Math, Magic & MTF:

A Cheat Sheet For The Vision System Community

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The Math, Magic & MTF: A Cheat Sheet For The Vision System Community

The best indicator of lens performance — what every buyer must consider when deciding if a specific lens will meet their vision-system needs — is found in a single number: the lens' *modulation transfer function*, better known as MTF. **But there's a problem.**

While MTF is the best proxy for lens performance, most optics professionals don't really understand what's behind the number, and we say that based on more than 35 years of experience working directly with them.

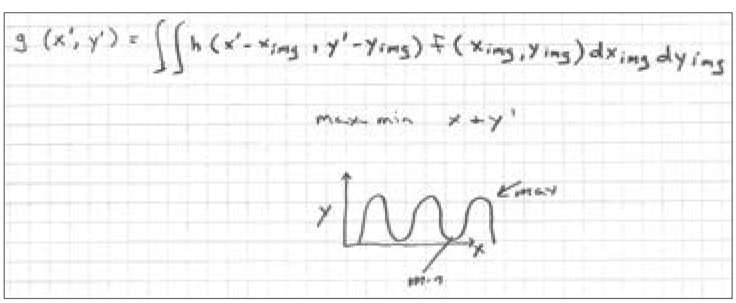
And there's no shame in it. The math behind MTF is beyond scary, the graphs used to plot it are often confusing, and the absence of standard terminology to describe it is misleading.

It doesn't have to be that way. And you don't have to be a mathematical genius to really "get" MTF. Let this paper be your guide.

Getting To Know MTF: Theory And Practice

Before we get to MTF, let's start with some fundamental truths in optics. First, the reason we have a lens is to reproduce an object into an image. And an object is made up of an infinite amount of small points. It is the job of the lens to re-image this multitude of points onto a sensor.

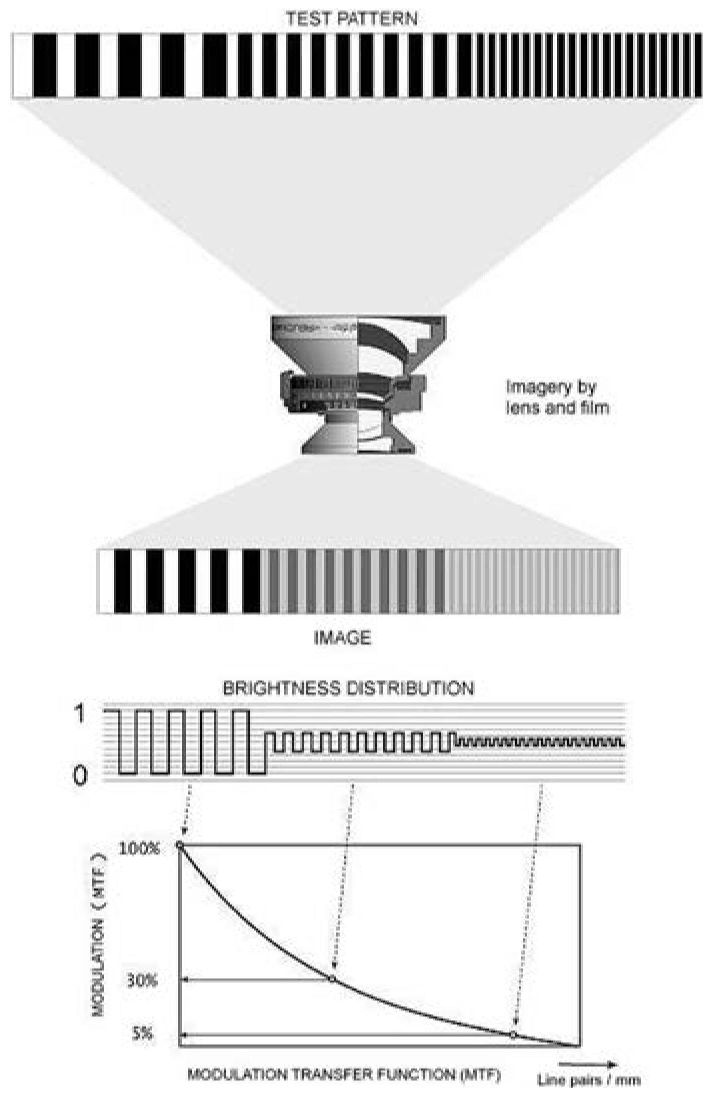
The image formed from each source point, or object point, will be located at their corresponding image points and weighted by the brightness of the original object at those locations to yield a continuous image function called "g."



Alright, we can hear you already, "Stu and Jim, you told me you were going to make this easy for me. This is not easy."

Bear with us. This will get easier soon. The long mathematical formula is as opaque as we'll get. And we'll confess, the math behind calculating MTF is as hard as any we have encountered. But let us show you what this formula really represents and the phenomenon it is actually describing, and you'll begin to see MTF more clearly for what it really is and what it can help you do.

Figure A: Object - Lens - Image example



This graphic is a test pattern and starts with very wide black and white bars. The bars begin perfectly black and white — a high-contrast image — but as you look from left to right, the bars get thinner and thinner as the frequency increases.

The lens looks at this pattern and reproduces it (see "IMAGE"). Notice how the big bars — considered low-resolution — remain black and white below the lens. The lens transfers them very easily. Yet as the frequency of the test pattern increases and the lines get smaller and smaller, the image values become grayer with less contrast between them. You no longer have perfectly black and white.

Instead you are getting medium shades of gray progressing to a point where you can barely distinguish the white from the black.

This phenomenon is described mathematically in the "brightness distribution" section of the diagram, where black is valued at 1 and white 0. Notice how the brightness decreases at the medium contrast as the image grays. Then all the way to the right, see how small the brightness distribution becomes: You almost can't distinguish the bright from the dark.

This conveys that a lens can transfer low-frequency, or low-resolution, images more easily than high, which should come as no surprise. Even a relatively inexpensive lens can transfer big bars or dots to form a pretty good image. Where a good lens shows its worth is at the higher and medium resolutions.

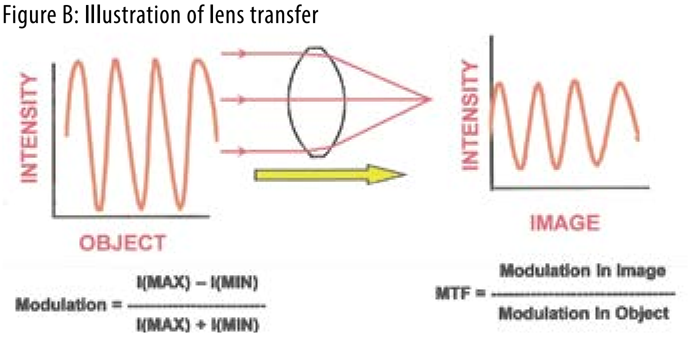
Now let's bring this home by examining the brightness distribution plot at the bottom. This shows how the brightness distribution modulation we just examined can be plotted. And guess what? This is what a typical MTF curve looks like. All MTF curves have the same shape — this shape — because every lens theoretically has a cut-off point where there is no modulation, where it can no longer produce an image. The plot tells you how well a particular lens reproduced its image.

So, while we started off with very complex math, we have quickly found graphical ways to represent that math that are much more palatable and a lot more illuminating. Simply put, we can tuck away the math and know there is a very easy way of taking a test pattern of an object, quantifying the image brightness, and graphing the modulation transfer function in line pairs per millimeter, which gets us to MTF.

Or at least most of the way.

Getting Dangerous

Figure B: Illustration of lens transfer



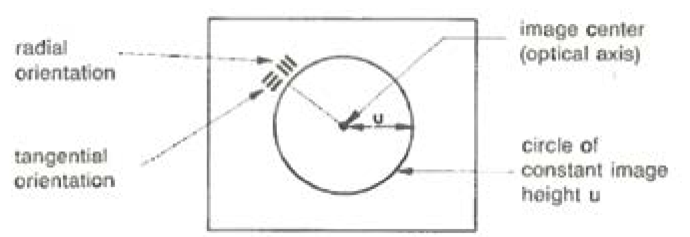
Now consider Figure B, which helps fill out the MTF story even further. To the left of the lens is an intensity profile of the object, which correlates to the perfectly black and white bars of Figure A. As the lens reproduces the object as an image, moving to the right across the graphic, the intensity of the black and white bars diminishes. This phenomenon is represented mathematically by the equations at the bottom. The first shows how to calculate modulation of the object and its image by using the maximum and minimum intensity values of each — the peaks and valleys of their respective troughs — and plugging them in accordingly.

The second reveals the golden formula for calculating MTF, one we can all handle: Dividing the image's modulation by that of the object. Which is simply to say that MTF is nothing more than the ratio of the modulation in image space to modulation in object space. Or, put a different way, MTF tells us how well the modulation in an object is transferred to an image by the optics.

Now you're dangerous.

But we want you to be *fully armed* when it's time for you to choose the right lens for your vision system, so let's look at Figure C.

Figure C: Radial and tangential orientation of object patterns



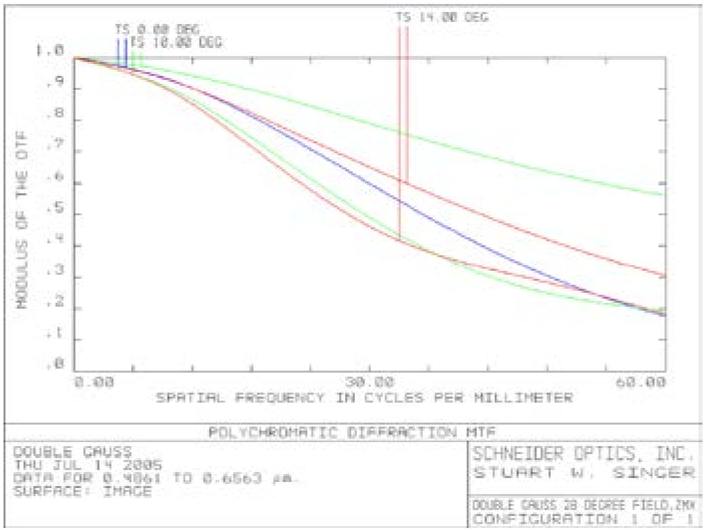
MTF In The Real World

MTF doesn't exist in a vacuum. It is impacted by the orientation of the object in space. Think of a machine-vision system looking for defects on a roll of tape — the defects can be going up at an angle, or horizontal or vertical to the camera.

Real MTF plots account for these factors, and they do so by describing MTF radially and tangentially. Together radial and tangential orientations provide MTF values across a 90-degree field of view, helping optical engineers avoid selecting a lens that performs well in one direction, but poorly in another.

This holistic view of MTF, representing structures that are perpendicular and tangential to the optical axis of the lens, is pictorially captured in Figure D, otherwise known as a classic MTF plot.

Figure D: Classic MTF Plot



How To Read A Classic MTF Plot

Optical engineers will recognize this classic MTF plot. It is what they get from a typical data sheet when dealing with a reputable optical company in North America. We mention North America specifically, because, as we've noted in past articles, our industry is rife with inconsistencies. Case in point, MTF is plotted differently in Europe and Asia than it is in North America, which is the subject of Figure E.

But let's dwell first on Figure D and understand what we are viewing. In North America, we have an xy plot with MTF always on the vertical y-axis. Here MTF is labeled "modulus of OTF," where OTF = optical transfer function. Don't be thrown off by this terminology switch: This is just a fancy way of saying MTF.

On the horizontal x-axis, we have frequency in line pairs per millimeter. So, here, we have a plot that goes from zero, where we would find theoretically huge black bars, all the way up to 60 line pairs per millimeter, which is technically 120 lines of alternating black and white bars in a tiny 1-millimeter space.

Now if you look in the upper left of the graph, you'll see the descriptor "TS 0.00 DEG," where the letter T is short for tangential, the letter S short for sagittal (frustratingly another word for radial), and DEG short for degrees. If you follow the two lines from 0 degrees, they point to a dark blue line, and it's only one single line — that's what we mean by "on axis." There's no split, no difference in radial or tangential. We're at 0.0 degrees, which represents the optical axis of the lens.

T and S, on the other hand, are considered an "off-axis" aberration, showing performance differences 90 degrees from one another. This performance difference begins to show the moment you leave the optical axis, or 0.0 degrees.

After TS 0 degrees, you'll find TS 10 degrees. There, one green line goes very high: The very top line drawn on this graph. It has an MTF value of ~58% modulation. That's the sagittal value — it has a lot of modulation in it, almost 60%. And the tangential value is at 20%.

The wide delta between the sagittal and tangential values shows something important: It means the lens has *astigmatism* in it. There is markedly better performance in one direction than in another at a single focus position.

That's a very handy thing you can extrapolate from an MTF plot. A substantial difference between radial/sagittal and tangential values — greater than 2:1 — implies the lens has astigmatism and might not be appropriate for your imaging system. Moving forward, consider this a dependable rule of thumb.

Now if you look at the last set of lines, which says 14 degrees, you'll notice the bottom one ending at 20% modulation, while the top one ends at 30%. This is telling us that as we go further off axis the lens doesn't have as much astigmatism, only 10%, which is perfectly acceptable.

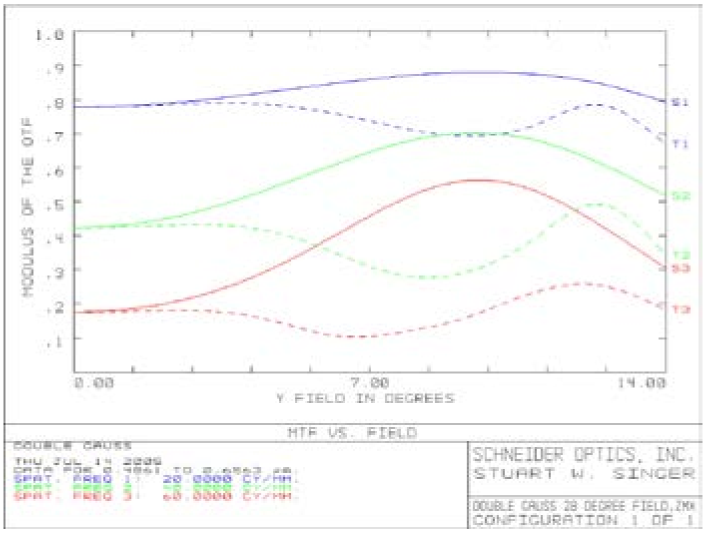
In summary, this classic MTF plot shows us how well this lens performs out to a set frequency or set amount of line pairs per millimeter. It tells us how well the lens can reproduce its object via the MTF. The higher the MTF, the better it can reproduce the object. The lower the MTF, the worse it can reproduce its object.

One last thing to say here about Figure D: Optical engineers always work in halves. When we plot MTF, we plot it with respect to either image plane height (a topic for another paper) or the angular field of view. And while we've plotted it here to an angular field of view of 10 degrees and 14 degrees, we are really doing so to + or -10 degrees and + or -14 degrees. In short, this MTF plot covers a full 28-degree field of view. This is due to lenses being considered rotationally symmetrical.

Through a Funhouse Mirror

Now onto Graphic E.

Figure E: MTF vs. image height



This is the way the international standard organization and German manufacturers typically plot MTF. Looks a lot different, doesn't it? Ironically, it is the same plot. The data is identical.

Here's how to take the confusion out of it.

On the vertical y-axis, where it says "modulus of the OTF from 0 to 1," that's the MTF. And notice on the horizontal axis where it says "Y FIELD IN DEGREES?" On our classic North American MTF plot, the x-axis showed frequency. Here instead the parameter is the field of view — exactly the same as what we measured before — all the way out to + or -14 degrees.

Let's look at some of the line pairs. The top lines are black, the middle lines green. The top lines start on the vertical y-axis at about 78%. Notice how they split as they move off-axis. One's a solid line, one becomes a dashed line: That's your radial and tangential.

Move on to the green pair, which starts at about 42%. Once you get out about 1 degree on the horizontal axis, it starts breaking up — see the dashes? And look at ~8 or 9 degrees. That big belly between the two of them highlights astigmatism.

The major difference between the European MTF plotting standard and that of North America is the Europeans plot the MTF using either the "field of view" or "actual image plane height." And the lines that are drawn within are discrete frequencies, and they're listed in the ledger below — 20, 40, 60 line pairs per millimeter. For an optical engineer's purposes, it's the same information as on the other plot, just shown a different way.

That said, we still have to ask why they have advanced this approach to plotting MTF. It shows *less* data. This plot only shows three discrete frequencies, whereas the classic North American MTF plot shows every frequency between 0 and 60 mm. Yet I have to admit, I do see the ease of their approach. It is handy to simply scan across the plot and see how lens performance changes from on-axis to extreme off-axis.

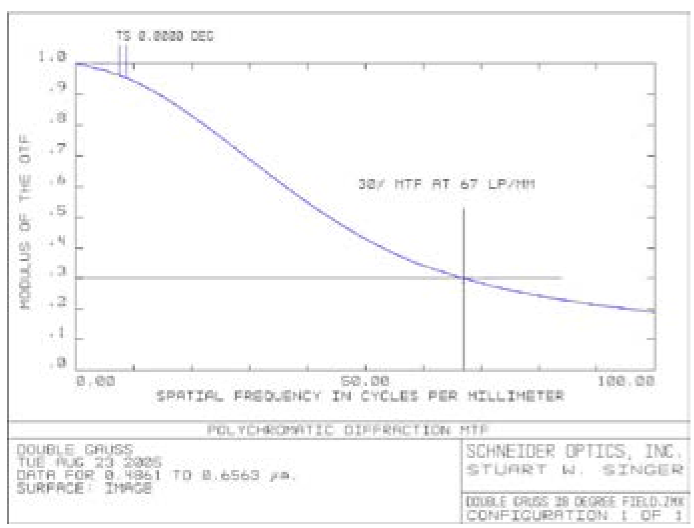
The Knockout Question
(And How To Use MTF To Answer It)

So we've covered a lot. We've looked closely at a number of MTF plots and equations, making them easier to read and navigate, while hopefully demystifying them in the process.

And that's great, but we know you have one more question, and it's a big one, *the big one*: How do I know if the lens I am looking at is going to be good enough for my system?

That's the knockout punch for MTF, and that's what Figure F delivers.

Figure F:



Here's another rule of thumb you can count on: Pick a lens with 30% contrast (MTF) at 2/3 Nyquist, and your lens will never be the gating factor of your vision system.

Let's break this down into manageable pieces. First, 30% MTF. We want to make sure our lens — no matter what field of view we are looking at, on-axis or off, has 30% or greater modulation three-quarters of the way out of the limit of its sensor.

The resolution limit of a sensor is called "Nyquist." And it can be calculated quite easily. Nyquist, which is always calculated in line pairs per millimeter and is nothing more than 2x the pixel size (in mm) divided into 1, so 1 over 2x pixel size.

But the pixel size has to be in millimeters, so know going into this that you'll have to make a quick conversion. Data sheets often list pixel size in micron. Just multiply the micron value by .001 (or just put two zeros in front of the number) and you'll have your conversion. A 5-micron pixel? .005 millimeters. Automatic!

So let's keep going with our 5-micron pixel to get to Nyquist. We already know our pixel size is .005 millimeter. We then multiply .005 by 2, and divide it into 1, resulting in 100 line pairs per millimeter.

So in Figure F, we have an MTF plot drawn out to 100 line pairs per millimeter, which represents a pixel size of 5 micron. And we want to ensure the lens has 30%t MTF or greater at Nyquist, which is two-thirds of 100 in this example, or 66.7. See the nice horizontal line coming across?

Don't get distracted by what is happening on the far right of the graph. Look away from 100! You want to stay focused on two-thirds of the limiting resolution. And you want to ask yourself: Does the lens have 30% or more MTF on- and off-axis? If you have that condition, you have the right lens, one that will not impede the performance of your system.

But here's a final tip: If you go all the way to 100 line pairs per mm, see how this lens ends at ~19% MTF? That's good. You don't want 30% or greater beyond Nyquist, because that can possibly bring Moiré effects into play. Ever see a news reporter on high-definition TV whose striped tie is blotting the screen — that's the Moiré effect. And it happens when the lens is simply too good. You don't want that, so remember to look at MTF at Nyquist, not 100%.

MTF Ninja!

We've covered a lot of ground here, moving from the powerhouse math behind MTF through the nuances of the curves that define its plots and, ultimately, its applicability to your lens selection and system design. Whether you need a refresher on Nyquist, a Rosetta Stone for translating international MTF plots, or a cheat sheet for "all of the above," keep this paper handy, because MTF isn't going away any time soon.

Please keep in mind that any lens will have different optical performance (MTF) at different aperture setting (f-numbers). Also the same is true for different working distance or magnifications.

So, when evaluating a lens you need to try to obtain an MTF Plot that closely matches your imaging requirements.

In fact, MTF has a positive cascade effect throughout vision-system design. Not only will your MTF acumen help you select the best lens for your system each and every time, but it can also help you calculate your system's total signal-to-noise ratio, which, by the way, is the product of each component's individual MTF.

That math is easy. Happy multiplying.

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