Theory and Measurement of the Modulation Transfer Function (MTF)

Table of Contents

1 The Modulation Transfer Function (MTF)

1.1 Introduction

MTF is a measure of the quality of an image formed by a lens or an electro-optical system. It has the following characteristics:

- It is a quantitative and objective measure.
- It can be measured with high accuracy and repeatability.
- It is universally accepted and understood.
- It can be calculated from lens design data, enabling actual and theoretical performances to be compared.
- It is appropriate for lenses of all qualities, from high-performance satellite camera lenses to low-performance lenses in web-cameras and mobile phones.
- It completely characterises the image quality in the measured region of the image.
- When one system forms an image produced by another, the MTF's can, under certain conditions, be multiplied together to obtain the overall MTF. For example, the MTF of a lens and the MTF of a sensor can be multiplied together to produce the overall MTF of a video or digital camera.

The measurement of MTF has been made much easier in recent years by the availability of high-quality video cameras and powerful desktop computers. Consequently, MTF measurement has almost completely replaced the older visual tests based on resolution charts, which relied on human judgement and provided only very limited information.

1.2 Image Formation

Any general object may be considered as a collection of point sources of light. In order to understand how a lens forms an image of such an object, we first need to consider how an individual point of light is imaged.

Figures 1 and 2 show a lens forming the image of a point object. In Figure 1 the object is at infinity, therefore the rays of light that enter the lens are effectively parallel, and they come to a focus in the plane which contains the back focal point, F'. In Figure 2 the object is at a finite distance from the lens, and the image plane is further from the lens than in the infinite conjugate case.

Figure 1

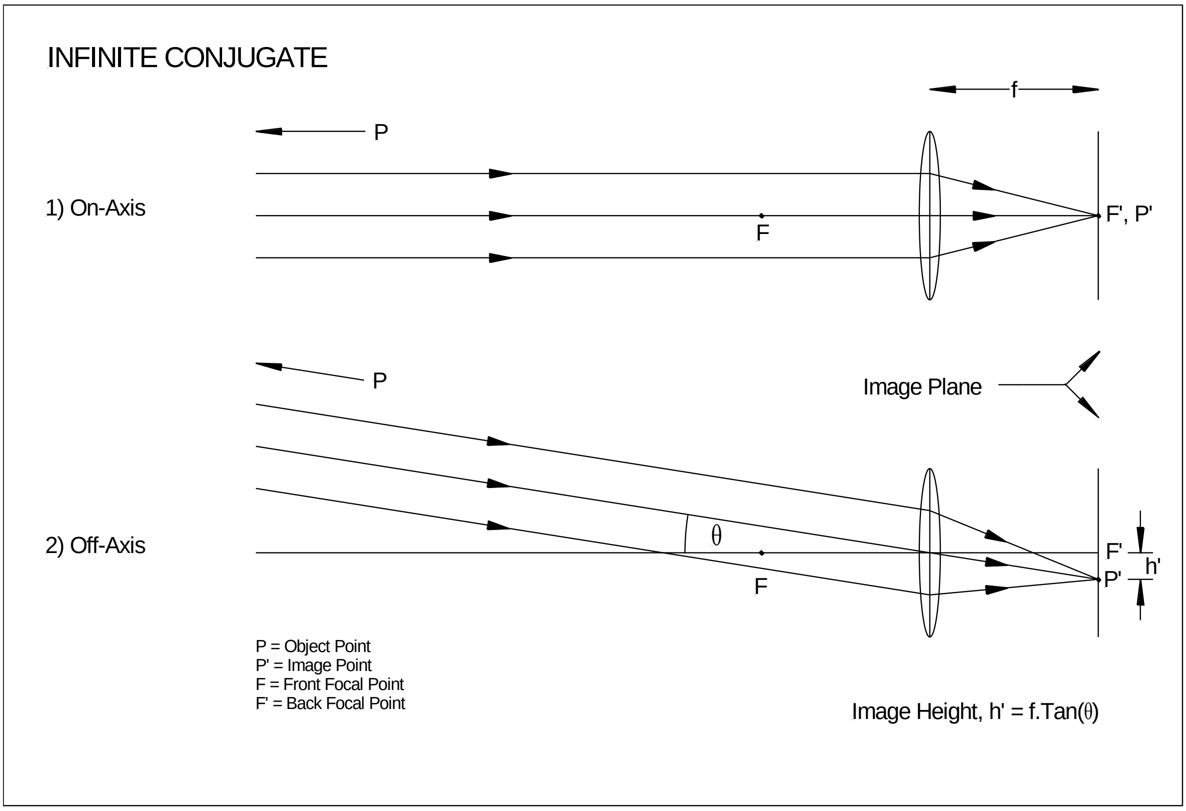
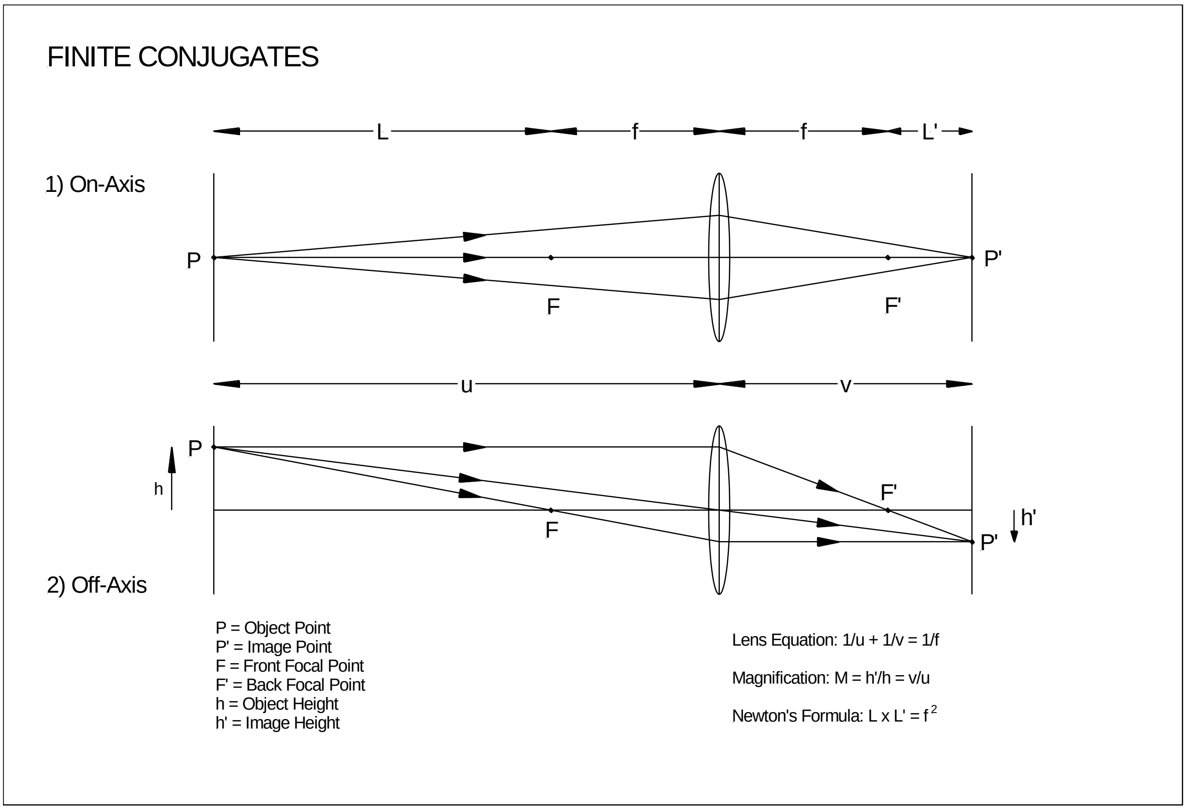


Figure 2



The spot of light in the image plane corresponding to the object point has a finite size and is called the Point Spread Function. It is the size and shape of this spot which will determine the quality of the image formed at the focal plane. If this spot is large, then fine detail in the image will merge together and be lost. A small image spot will retain more detail in the image.

The factors which determine the size and distribution of light within the Point Spread Function are:

- Diffraction

  If all the rays converging on the image point intersected at a single point, the PSF takes the form of a small disk, surrounded by weak ‘diffraction rings’. This is called the Airy disk. The diameter of the Airy disc reduces as the aperture of the lens increases, but nevertheless it remains finite. Therefore, even a perfect lens produces an imperfect image.

- Lens Aberrations

  Rays of light leaving a point in the object do not generally meet at a single point in the image. This is called aberration, and generally becomes progressively worse from the centre to the edge of the image plane. Lens designers make every effort to minimise aberration as much as possible, but they constrained both by the laws of refraction and the manufacturing budget for the lens. The spread of rays around the ideal image point tends to enlarge and distort the Airy pattern, often beyond all recognition, and reduce image quality further compared to the “diffraction-limited” case.

- Manufacturing Errors

  A real lens will never conform exactly to the design parameters. The surfaces will deviate to some extent from the required curvatures, and there will be errors in the centring and positioning of the individual elements. The refractive indices of the elements may also differ slightly from the expected values. The consequence of all these errors in combination is usually to disperse the rays in the image spot even further, enlarging the blur spot pattern and reducing image quality.

Therefore, the distribution of light in an image formed by a lens always differs from that in the original object. In general there is a loss of detail, that is a blurring or smoothing of the original scene. This happens because a point in the object is not imaged as a perfect point in the image, but instead as a Point Spread Function of finite size.

The size and form of the image spot (PSF) changes with position in the image plane. Near the optical axis the spot tends to be relatively small and circular, whereas in the corner of the image plane it tends to become larger and more complex in shape. It is obvious that the quality of the image will depend in some way on the size and shape of the PSF, and in general the smaller the spot the better. However, it is not so obvious how to derive a simple and meaningful measure of image quality from the often-complex form of the Point Spread Function. The concept of MTF was developed to provide a solution to this problem.

1.3 The Sine-Wave Target

The concept of MTF is most easily understood with reference to a sine-wave target. A sine-wave target consists of a regular pattern of black and white lines, where the variation from black to white follows a sine-wave pattern (see figure 3).

Figure 3 – A Sine Wave Target

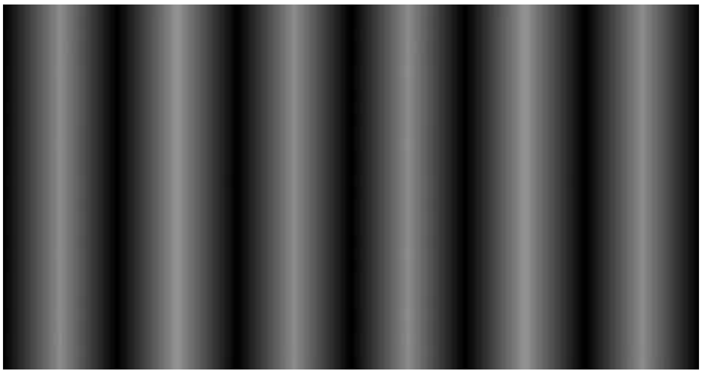
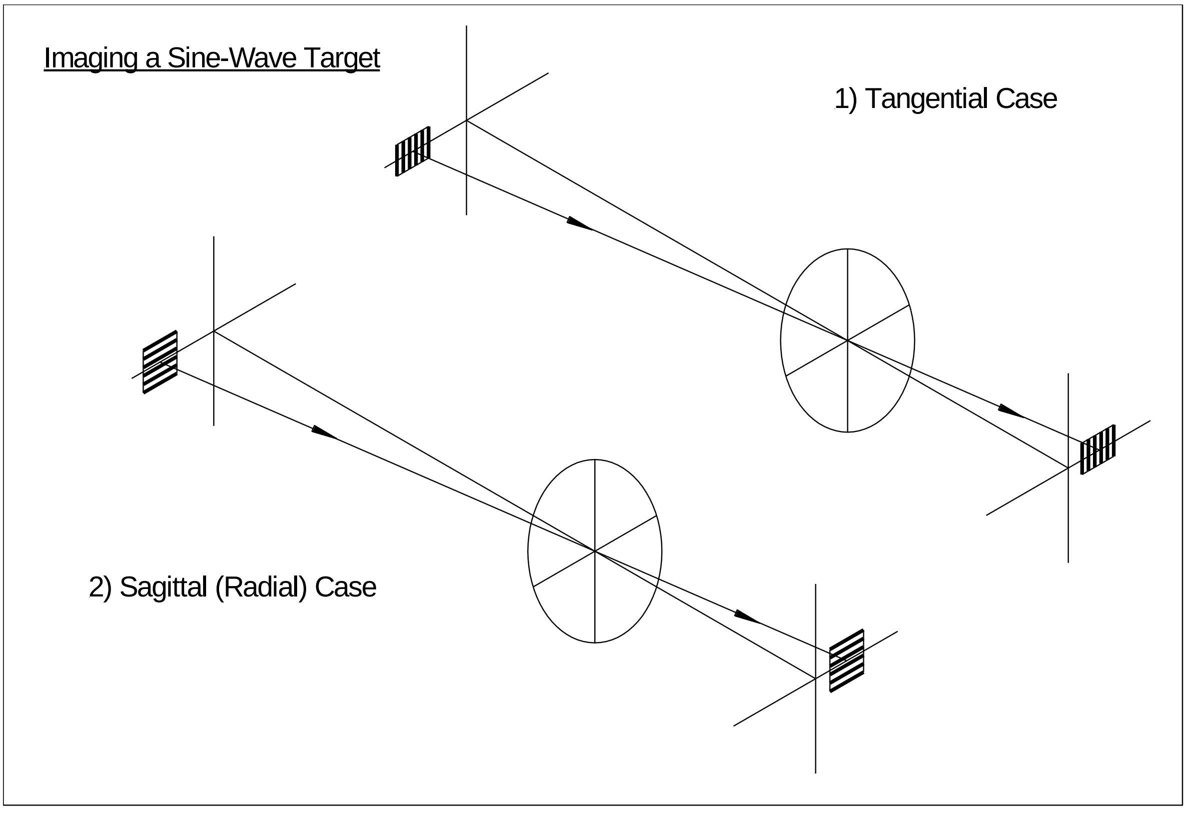


Figure 4 shows a sine-wave target being imaged by a lens. The target is centred on an off-axis object point. In the first diagram the lines of the target are parallel to the tangent of the circle centred on the optical axis – this is called the Tangential orientation. In the second diagram the lines of the target are parallel to radius of the circle centred on the optical axis – this is called the Radial, or Sagittal, orientation.

Figure 4



The most important characteristic of a sine-wave target is that it is always imaged as a sine-wave pattern, no matter how complex the Point Spread Function happens to be (provided the PSF is more-or-less constant over the target area). In general there will be a reduction in the contrast of the image, because light from the bright areas of the pattern will fall into the dark areas, due to the finite size of the image spot. This reduces the range of brightness, or contrast. If the distance between the bars is large compared with the image spot size, the contrast reduction will be small, but when the distance is comparable to the blur spot size, the contrast reduction becomes much larger. In fact, there is a line spacing at which the contrast in the image becomes absolutely zero - the image is just a uniform patch of grey, and it remains so for all targets whose lines are more finely spaced.

1.4 Definition of MTF

We will now proceed to define some terms required to define MTF. Figure 5 shows a graph of the variation of intensity in a sine-wave target with distance. The target is characterised by its Spatial Frequency and Contrast.

Spatial Frequency (u) is the number of cycles (or line-pairs) per millimetre, and is given by

u = 1 / P

where P is the period of the wave (the distance between crests) in millimetres.

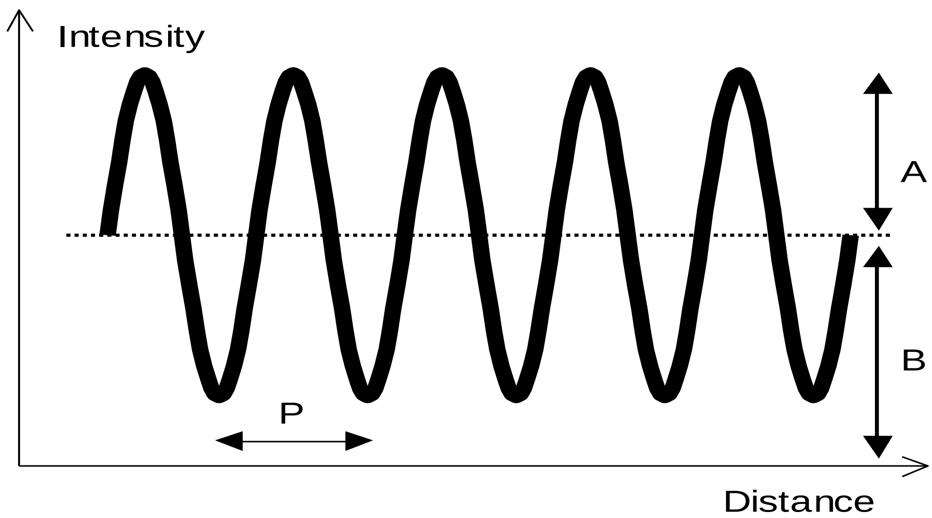


Figure 1 Intensity variation in a sine-wave target

Contrast is the ratio of the amplitude of the sine-wave, A, to its average value, B. That is

C = A / B

Since,

A = (Max – Min) / 2

and

B = (Max + Min) /2

Then the contrast can also be defined as:

C = (Max – Min) / (Max + Min)

Note that C must lie in the range 0 to 1. If the contrast is 1 the centres of the black lines are completely black. If the contrast is 0 the target is uniformly grey.

Now we can define the MTF of an imaging system. The MTF, or Modulation Transfer Function, is defined as the ratio of the image contrast to the target contrast, expressed as a function of spatial frequency. That is,

MTF(u) = C'(u) / C(u).

C is the contrast in the target, C' is the corresponding contrast in the image.

For low spatial frequencies the MTF is nearly 1.0 or 100%. The curve then generally falls as spatial frequency increases, until it reaches zero, the limit-of-resolution for the lens. Test patterns of this frequency and above are imaged with zero contrast, that is, as a patch of uniformly grey light. Figure 6 shows a typical MTF curve.

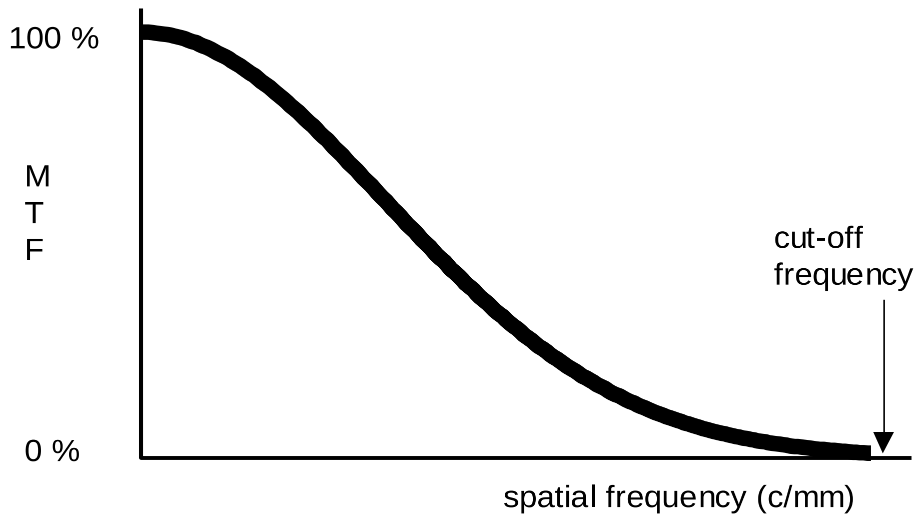


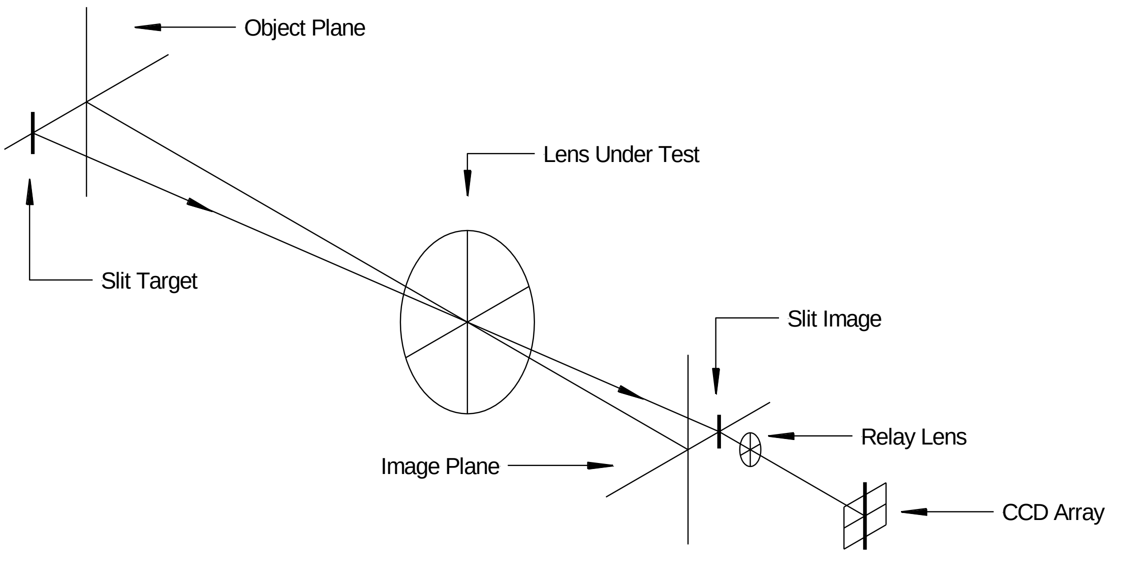
Figure 6. An MTF Curve.

The MTF curve plots the contrast in the image of a sine-wave target as a function of spatial frequency. It applies to a particular image point and target orientation (Tangential or Radial). In order to characterise a lens fully MTF measurements must be made both on-axis and at a number of points in the field, and at each point it is usual to measure the MTF in both T and R orientations.

1.5 MTF Measurement

In practice, MTF can be measured by presenting a series of sine-wave targets of increasing spatial frequency to the system under test, and then measuring the contrast in the corresponding image patterns. Plotting the ratio of image contrast to target contrast against spatial frequency will then produce the MTF curve.

However, such a method would be very slow and laborious, and today practical measurements are usually made using point, line, or edge targets. These methods have been made possible by the arrival of the modern computer. Figure 7 shows the measurement of MTF using a slit target.



MTF Measurement using a Slit Target

Figure 7

The variation of light intensity across the image of slit target is called the Line Spread Function, or LSF. It can be shown that the MTF, which is defined in terms of sine-wave targets, is actually the Fourier Transform of the LSF curve. Once the LSF measured, the MTF can be calculated in just a few milliseconds using a modern desktop computer.

In practice the width of the slit used for MTF measurement must be small compared to the width of the PSF. The variation of light intensity across the image can be measured using a light detector with a small sensing aperture. The detector is moved mechanically in small steps across the image, and the resulting intensity profile is the LSF of the lens under test.

A more convenient method employs a CCD detector array to measure the LSF data. It is usually necessary to enlarge the image first using a relay lens (typically a microscope objective) before the image can be analysed.

Figure 8. Slit Image from CCD Camera

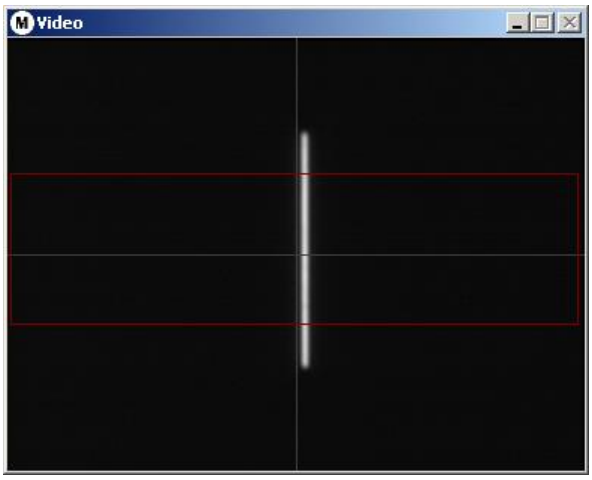


Figure 9. Line Spread Function (LSF)

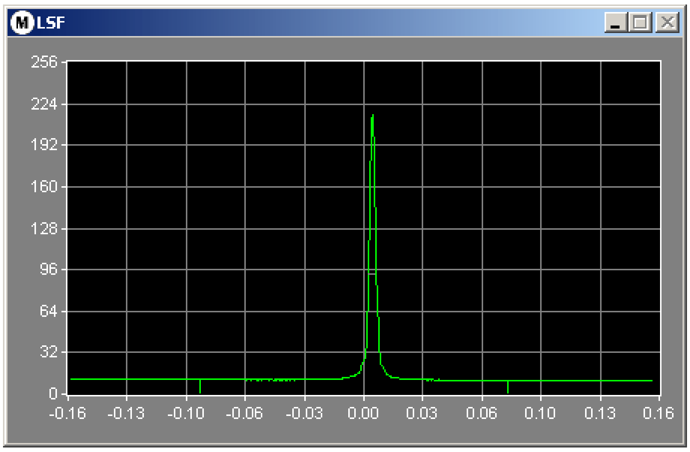


Figure 10. Modulation Transfer Function (MTF)

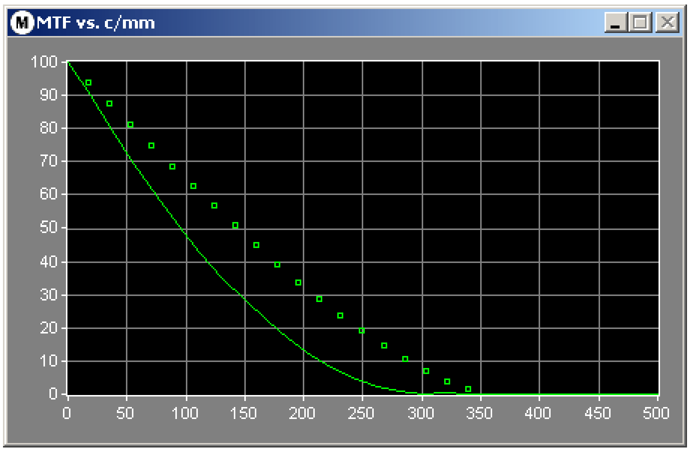


Figure 8 show the slit image on the CCD camera. Figure 9 shows the corresponding LSF curve, which plots the average pixel value in each column against horizontal distance. Figure 10 show the MTF curve calculated from the LSF in figure 9 using a Fourier Transform procedure (the individual points show the diffraction-limited MTF curve in this case).

1.6 Characterising a Lens

Lens testing would be a very simple procedure if the size and shape of the Point Spread Function were constant over the field-of-view of the lens. A single measurement in the centre of the field would then fully characterise the lens. However, this is unfortunately not the case, and the variation of the PSF over the field is usually considerable. Therefore, in order to characterise a lens completely, the MTF measurement must be carried out at several different field points, from on-axis and out to each corner of the field. This is usually achieved by moving a single test target to different positions in the field sequentially, and moving the detector to the corresponding image positions to analyse the images formed. If measurement time is paramount, sometimes multiple targets and detectors are used in order to reduce the number of movements required.

Another complication arises from the fact the MTF depends on the orientation of the target. Fortunately, it is usually sufficient in practice to check only two orientations, the Radial and Tangential orientations. In the Radial orientation the lines of the test target are parallel to the radial line, which is the line drawn from the centre of the field to the centre of the target. In the Tangential orientation the lines of the test target are perpendicular to the radial line. These two orientations (or azimuths) generally provide the best case and worst case MTF's respectively, which justifies measuring in only these orientations.

A further complication arises from the fact that the form of the PSF, and hence the MTF, also depends on the colour of the light. Generally colour filters are used to modify the spectral response of the system (light source output and detector sensitivity) to match that of the system in which the lens will eventually be used. Sometimes it is desirable to measure the MTF with different coloured filters so that the image quality can be assessed independently for each colour band.

Lastly, the MTF is sometimes measured at different focal planes, so that the manufacturer can check that the lens will meet the specification even when imperfectly focussed.

It can be appreciated from the above that the full characterisation of a lens can involve a large number of MTF measurements. For example, suppose that a lens is to be measured on-axis and at 6 points in the field. Measurements will be made in both Radial and Tangential directions, and repeated for three different colours - Red, Green, and Blue. That is a total of 7 x 2 x 3 = 42 MTF curves. This number of measurements is not unusual today and is the reason why computer-controlled automated test benches are being widely adopted.

