Using The Bessel Null Method To Verify FM Deviation Measurements

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Frequency modulation (FM) has been used in various radio frequency (RF) transmitters and receivers for decades. While more complex forms of modulation have been developed, analog FM is still in widespread use for walkie-talkies, police/fire/military radios, FM broadcast radio, telemetry, and other systems. Consequently, RF and microwave test equipment -- signal generators (sig gen), signal analyzers, and modulation analyzers -- often provide features to generate or measure FM.

FM modulates the RF carrier *frequency*. The frequency deviation (f_{DEV}) -- the peak frequency *change*, above and below the unmodulated carrier frequency – is a critical parameter of FM.

The question may arise: Just how accurately can an analyzer measure f_{DEV}? The **Bessel Null Method** is a simple way to verify f_{DEV} accuracy -- in a way that is ***not dependent*** on the f_{DEV} specs of any sig gen -- with excellent precision and high confidence.

Review of FM and Bessel Functions

FM modulates the ***frequency*** of the RF carrier. Ideally, the carrier's *amplitude* remains constant. The frequency deviation (f_{DEV}) is directly proportional to the instantaneous amplitude of the modulating signal.

In this paper, we consider the special case where the modulating signal (sometimes called audio, baseband, or tone) is a single, pure, symmetrical, sine waveform of constant amplitude. The frequency or rate of this audio tone will be called f_{RATE}.

We can see the RF spectrum of an FM-modulated signal on a spectrum analyzer (SA); see Figure 1. The central response corresponds to the carrier; then working outward, there are pairs of symmetrical sidebands, spaced at intervals of f_{RATE}. (Note: The resolution bandwidth (RBW) filter of the SA must be set narrow enough ($<f_{RATE}$) to resolve these sidebands.)

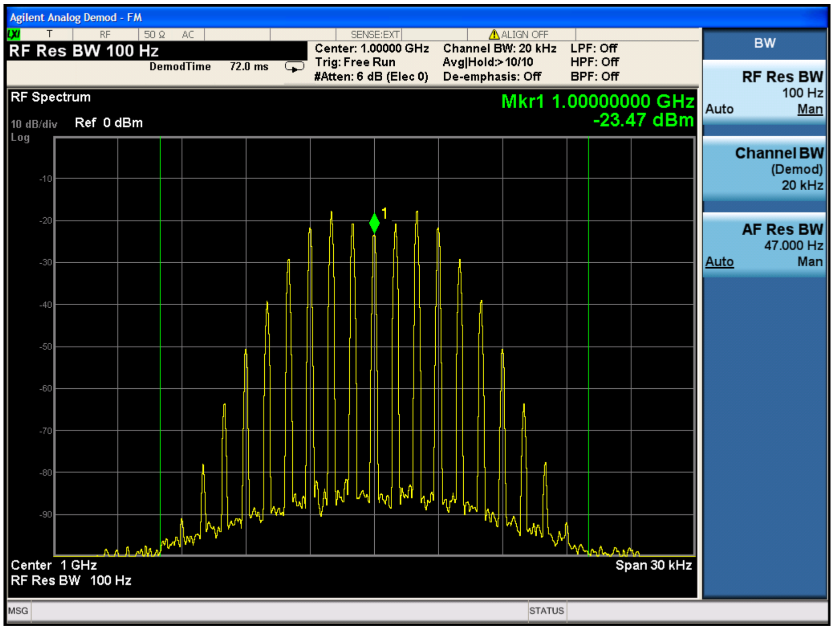

Figure 1: Example RF Spectrum for FM. For this example, f_{DEV} = 3 kHz peak and f_{RATE} = 1 kHz. The Res BW filter in the SA has been reduced to 100 Hz, to resolve the sidebands clearly.

The amplitudes of the sideband pairs are described by the Bessel function of the first kind; see Figure 2.

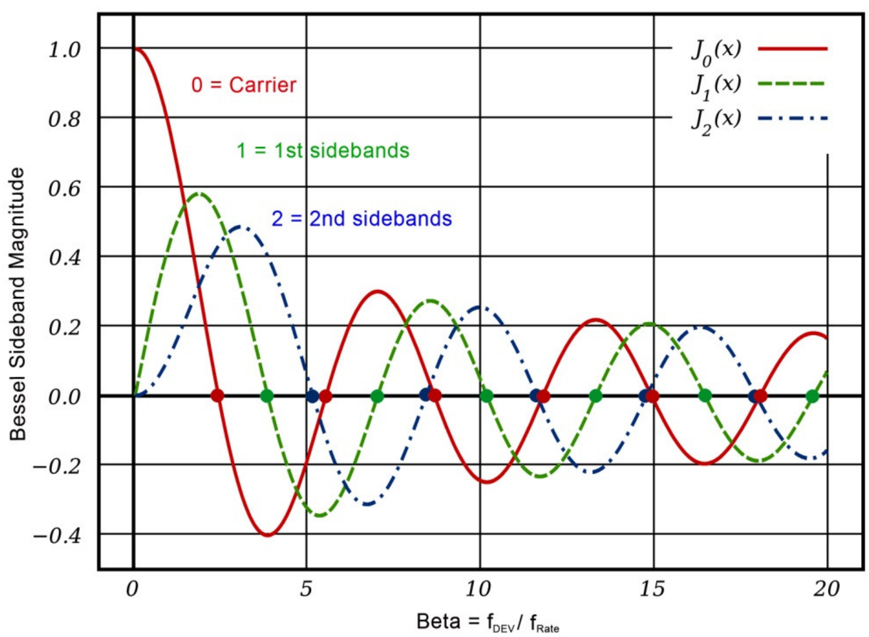

Figure 2: Bessel Functions. The dots emphasize the nulls.

The horizontal axis is β (beta), where $\beta = f_{DEV} / f_{RATE}$. If f_{RATE} is constant, then the horizontal axis is proportional to f_{DEV}.

The vertical axis is the relative amplitude of sidebands. The J_0 line describes the amplitude of the *carrier* (the 0th sideband), as a function of β. The J_1 line describes the amplitude of the *first* pair of (adjacent) sidebands, as a function of β. The J_2 line describes the amplitude of the *second* pair of sidebands. And so on.

Negative Bessel values represent a phase reversal; however, the SA displays magnitude only, so both positive and negative Bessel values appear as (positive) spectral responses. You can compare the amplitudes of the sidebands seen in Figure 1, with the values for curves in Figure 2, for β = 3.

As β varies (moving left or right), each Bessel curve passes through zero several times (emphasized with dots). Each of these is a "null", and corresponds to a value of β where the carrier (or a sideband pair), corresponding to that curve, has zero amplitude. The values of β at these nulls can be calculated exactly; see Table A. Recall that $f_{DEV} = \beta * f_{RATE}$. Since β at the nulls is known exactly, and f_{RATE} can be controlled or measured precisely, then we can know f_{DEV} at the nulls with great precision. This is the foundation of the Bessel Null Method.

Table A: Bessel Nulls [partial list, from Weisstein]

	β (beta) = f_{DEV} / f_{RATE}					
J_0 Carrier	2.4048	5.5201	8.6537	11.7915	14.9309	...
J_1 1st Sidebands	3.8317	7.0156	10.1735	13.3237	16.4706	...
J_2 2nd Sidebands	5.1356	8.4172	11.6198	14.7960	17.9598	...
...	...	...	...	...	...	...

Using the Bessel Null Method

Find an RF sig gen capable of high-quality (low distortion) analog FM. Ideally, it has a built-in audio function generator for modulation, and at least 1 Hz *resolution* in the settability of FM deviation (f_{DEV}). The *accuracy* specs for f_{DEV} are not critical. Set up the signal generator for the RF carrier frequency needed (e.g. 1 GHz), with RF on and adjusted to appropriate amplitude.

Choose a Bessel null that is near the conditions of interest. For example, narrow-band FM radios are commonly tested at f_{DEV} = 3 kHz and f_{RATE} = 1 kHz audio tone, or Beta = 3. We see in Table A that Beta = 2.4048 is the nearest null. So to get that null, we could choose

f_{DEV} = 3 kHz and adjust for $f_{RATE} = (f_{DEV})$/Beta = 1.248 kHz

... or ...

f_{RATE} = 1 kHz and adjust for $f_{DEV} = (f_{RATE})$ * Beta = 2.4048 kHz . We will use the latter.

Enable FM modulation. Provide the modulating sine tone at *exactly* 1.000 kHz. On the sig gen, set f_{DEV} to 2.4048 kHz (nominal). If the sig gen has 1 Hz resolution, use 2.405 kHz.

Connect the sig gen output to the SA. Set center frequency and span to center the RF spectrum. Adjust resolution BW (RBW) filter narrow enough to resolve the FM sidebands and reduce the noise floor. The spectrum should be symmetrical, and the central response (the carrier) should be fairly low, similar to Figure 3.

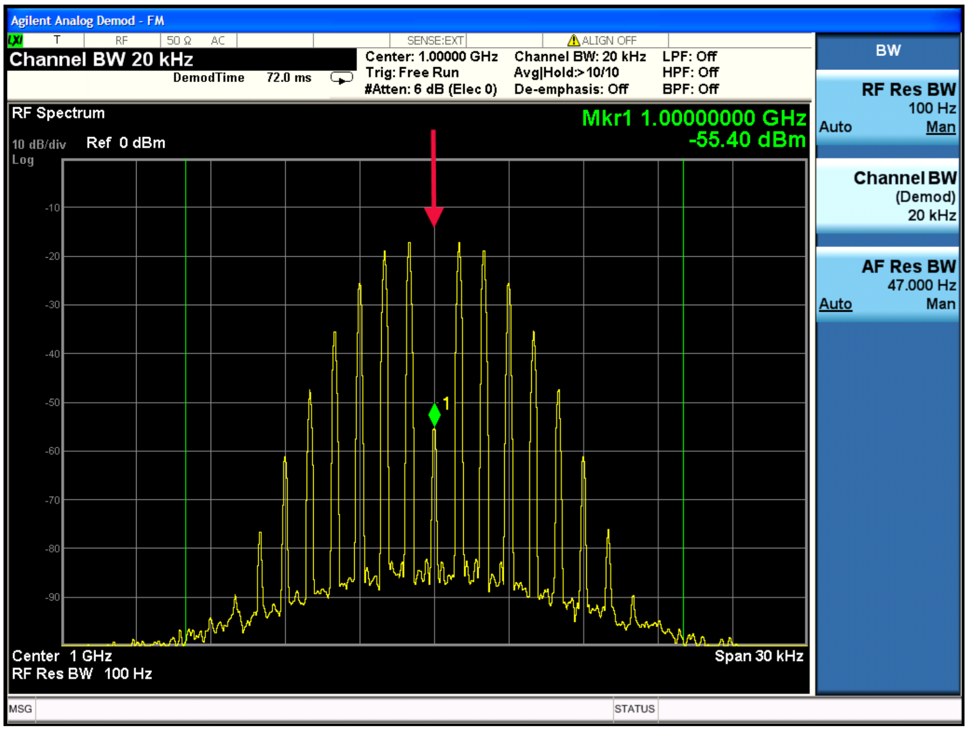

Figure 3: RF Spectrum near 1st J_0 Null; level of carrier (in the center) is > 30 dB down, compared to Figure 1.

Next, on the sig gen, slowly adjust f_{DEV} up or down in 1 Hz increments, until the central response (the RF carrier) is minimized. If necessary, reduce the SA RBW to further lower the noise floor. When the carrier is minimized, then f_{DEV} is 2.405 kHz (actual), *regardless of the* f_{DEV} *value indicated on the sig gen*. If a range of values appear to give equal nulls, use the middle value. See Figure 4, and compare to Figure 3. (You may or may not get a null this deep; just look for the minimum.)

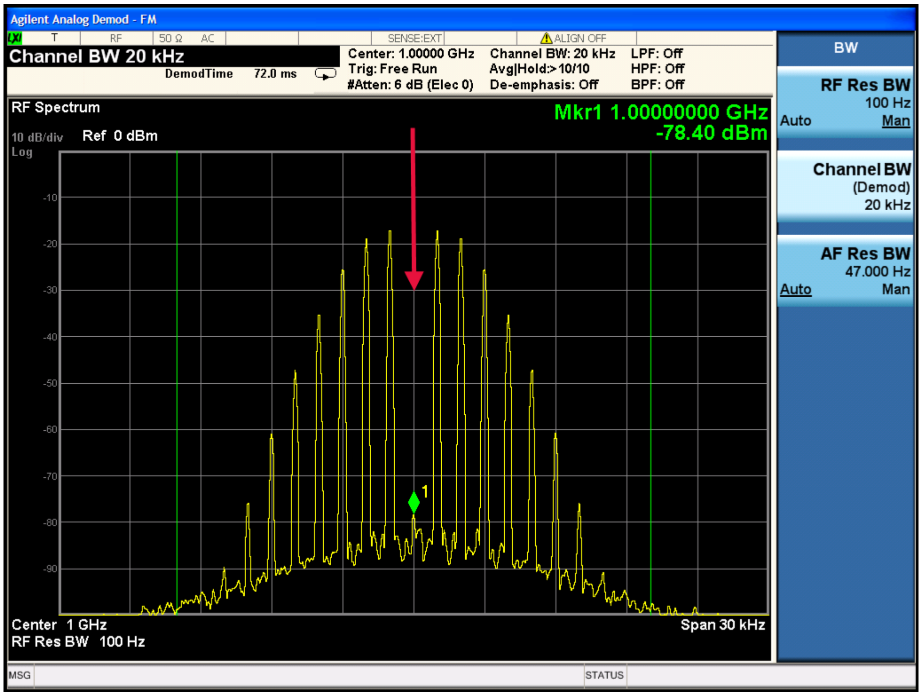

Figure 4: RF Spectrum with carried minimized, at 1st J_0 null. The frequency deviation where the carrier is deeply nulled can be precisely known.

To test an FM modulation analyzer, connect the sig gen to the analyzer's input -- *without changing the sig gen's settings* – and measure f_{DEV}. Any difference from 2.405 kHz is an error in the analyzer's f_{DEV} measurement.

This method can be used for other values of f_{RATE}; with any of several nulls (at various values of β); and for several sidebands (corresponding to J_1 or J_2 etc.). A related method uses Bessel functions but not nulls; instead, values of β are chosen where one pair of sidebands are known to have the *same* amplitude (non-zero) as *another* pair of sidebands. These variations make it possible to cover a wide range of f_{DEV} and f_{RATE}.

Discussion

How precise is this method? For a signal generator with high-quality internal FM, the primary limitation is the *resolution* (settability) and stability of f_{DEV} on the source. If the sig gen has 1 Hz resolution for f_{DEV}, the null and its f_{DEV} value can be found within ± 1 Hz resolution. In our example, that's 1 part in 2405, or < 0.05%. We are *not* dependent on the sig gen's f_{DEV} *accuracy* specification.

The specifications of the SA are *not* critical to find nulls; we are simply seeking a minimum.

The sig gen will have some level of phase noise (and possibly carrier leakage). This energy will limit the depth of the null, making it appear less sharp. Even so, the *center* of the null can still be estimated as the mid-point between equally low carrier levels near the null.

The remaining error sources are likely small or indirect contributors: distortion on the modulating tone or in the FM modulator [Broderick]; incidental AM; accuracy of f_{RATE}; plus noise and drift effects. Various metrologists [Lee, Skinner] estimate overall accuracy at < 0.15%.

A gross error, of course, is being on the wrong null. If this is a risk, then procedures must be devised to verify that the intended Bessel null is the one being used.

The Bessel Null Method has been used to verify and compare modulation analyzers. For example, the HP 8901 modulation analyzer and HP 8902 measuring receiver are often taken as the "gold standard" for measuring f_{DEV}. True, they set new levels of accuracy (1%), in their day. But the Bessel Null Method can demonstrate that modern solutions -- such as the Agilent X-Series signal analyzers with the N9063A analog demodulation measurement application (see Figure 5) – measure f_{DEV} even more accurately.

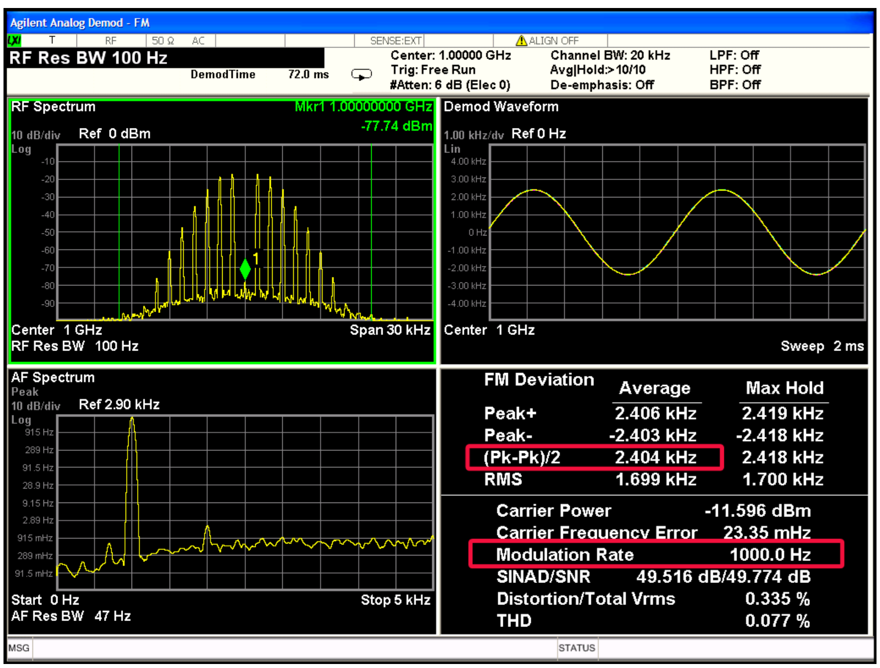

Figure 5: N9063A analog demodulation application. The RF spectrum (upper left) shows that the carrier is nulled, per our example at f_{RATE} = 1.000 kHz and f_{DEV} = 2.405 kHz. The measured FM deviation (lower right at (Pk-Pk)/2) reads 2.404 kHz, which is within < 0.1%.

Acknowledgements & References

Thanks to Jeff Nelson of Agilent Technologies.

Agilent “BenchVue” was used to capture and annotate screen images. Download this no-cost application at www.agilent.com/find/benchvue.

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